

Lawvere We think it is necessary to discuss these questions in a serious way.

People practicing practice a position on this point, and are prisoners of reactionary philosophy if they don't.

One part that should be discussed further, points raised in my talk in Durham / Milano last summer, under "Log. of Math  $\leftrightarrow$  Math. Logic."

Looking at notes people took:

Made some misformulation.

Did not choose words well

Difficulty of communication...

Cultural distortion: formulations were changed. more dogmatic formulation, for two different reasons.

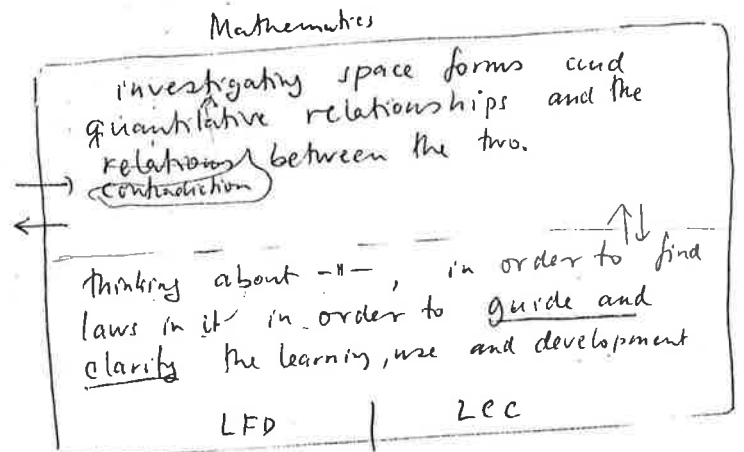
Leading idea in these talks:

There are certainly things that we are against. On the other hand, we cannot arrive at judgement on basis of deduction from some formulation, but only by sub...

We want to reject

"thinking about thinking" as an end in itself

obscure writing  
use of symbols to hide concepts



antagonistic contradiction (as trends)

Between these two we have non-antagonistic contradiction, but rather the contradiction leading to progress in science.

"Lower part" divides into

LFD short for Logic of Formal Deduction or Science of Formal Deduction of truth of properties.

Lee science of conceptual construction.

I referred to "upper part" as mathematics  
or the object of math (lower part then subject of  
mathematics.\* ) Nearly every piece of math involve ~~so~~ usually  
an advance in both.

I listed subjects that to a large measure are  
parts of subjects of math, not object of math.

- Abstr. alg. . .
- Gen. top . .
- Cat. theory . .
- Math. Log. . .
- Set theory . .
- Funct. anal. . .

They really should be one subject.  
 The fragmentation ~~sto~~ of these 6 should be eliminated.  
 At present time, cat. theory have subsumed these subjects  
 in so far as they belong to logic of math (not in so  
 far as they belong to math. in proper sense).  
 At same time, cat. theory is not identical with lower,  
 because . . may discover more principles.

Number theory?  
 [Bo Mazur: Yes: we use it to count space invariants.]

Unity, not an eclectic bag of tricks.

The science of thinking about math. (=logic of math)  
 must address itself both to LFD Lcc.  
 Mathematical logic is only LFD.

Object to Math. logic being the whole  
only this thing in principle, but as it exists, now.

Analysis of the enemy camp: ("counterpurpose")

\* or "the logic of mathematics!"

"Waving the flag of LFD in order to oppose

1) LCC, especially ...

2) the purpose (= acting as a clarifying ...).

Springs from one sidedness. Could also imagine the other one-sidedness LCC, but not the main danger at the moment).

Cat. theory is mainly LCC.

[Tierney: much of it is in the reject column] FWL: Yes.

In Math Logic the concept seem to drop from the sky. (LFD one-sidedness). Cat. theory viewpoint often sees the truth about ~~an object~~ a secondary this, the object / concept as primary.

The existence of Eucl. Geom. is more important than any specific Thm in it. ~~it~~ The fact that there must be a continuum is more important than the way you 'construct' it, or deduce...  $LFD \rightleftharpoons LCC$

Still: LFD is overestimated, LCC underestimated, we still have to fight for that, fight to show its importance.

Math. Logic vs. Conceptual science

which is the leading aspect? In a certain way have to claim Conceptual science is the 'leading aspect'. There could be no discussion on 'truth' or 'falsity' of addition, integration, and other concepts.

Shown by the fact that cat. theory includes logic. The 'theory' in the sense of cat. can be presented or represented by axioms and relations.

The contradiction  $LFD \rightleftharpoons LCC$  is a reflection of contradiction between math and logic of math.

sc. of deduction & calculation  
sc. of concept formation.

Theoretically  $\nearrow$  is more basic, but in the practice, it is  $\longleftarrow$

Logic of Math  $\neq$  Math.

although many people read they were equal (e.g. Brouwer, Bishop: space is nothing but nat. no's. i.e. nothing but thoughts).

In mathematics, the nat. no. does not exist as an object. It exists in logic of math. I agree with Brouwer that  $\mathbb{N}$  is subjective, in contrast to the continuum which is objective. Leads to "space filling curves" which do not exist. Cf. Arndt-Jensen paper. I agree with that position.

Space is here now

"All our possible future calculations are here" is not true.

Question of classifying 'toposes': it objectifies the subjective.

Mistakes I made in a paper: on quality and quantity.

Qualitative distinction between class topoi, and spatial topoi.

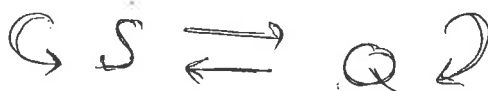
Construction of it depends (till now) on  $\mathbb{N}$ .

Guidelines to be drawn from this:

We should have the confidence that we can oppose obscure writing: have enough math knowledge and philosophical guidelines to get behind.

Reflect the whole picture into Lee by axioms?

Space Quantity: Four kinds of relations



Have scheme for a category: these commutative diagrams are defined. So unity "synthetic" and analytic geom.

Should lead drastically to simplification of highschool math, without using any quantifiers. Entirely Cartesian logic.

Process of clarifying Euclid's axiom has by no means been finished. Deepen this to diff eq's? Every object has a measure (length, area, ...) (non-measurable sets again only by  $\mathbb{N}$ ).

Consistent: we cannot out of it construct  $\mathbb{N}$ .

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The integral exists in the real world. In  $\mathbb{Q} \Rightarrow \mathbb{Q} \ni$   
should be postulated (not defined by an approximation  
process). That integrals can be approximated is a meta-  
Theorem.

Cal theory could serve directly in mathematics, without  
any reformulation via ...

Not only  $L_{ec}$  dominates over  $L_{FD}$ .

Dialectical logic will have to dominate over intuition  
Thanks to Chris, we know that lattice of subtopo..  
is dual of Heyting alg., so rather reject:

$$* \quad a \wedge \neg a \neq 0 = 0$$

Dialectical logic dominates over intuitionistic.  
[In dialectics  $*$  is not valid]

Joyal: Finishe methods in metamathematics.

But only formulae were really there.