Lectures on Quantum Groups : Corrections

The misprints marked † were corrected in the second printing. (I think the last number in the first sequence on the bottom line on the page opposite the Contents is the number of the printing.)

p. 7, 2nd displayed equation: The numerator should be $v^{a-b} - v^{-a-b} + v^{a+b} - v^{a-b}$

p. 7: Replace the last six lines by:

$$\begin{aligned}\sum_{i=0}^{r}(-1)^i v^{i(r-1)} \begin{bmatrix} r \\ i \end{bmatrix} &= \sum_{i=0}^{r}(-1)^i v^{i(r-1)} \left(v^{-i} \begin{bmatrix} r-1 \\ i \end{bmatrix} + v^{r-i} \begin{bmatrix} r-1 \\ i-1 \end{bmatrix}\right) \\ &= \sum_{i=0}^{r-1}(-1)^i v^{i((r-1)-1)} \begin{bmatrix} r-1 \\ i \end{bmatrix} \\ &\qquad - \sum_{j=0}^{r-1}(-1)^j v^{(j+1)(r-1)} v^{r-j-1} \begin{bmatrix} r-1 \\ j \end{bmatrix}\end{aligned}$$

where we interpret $\begin{bmatrix} r-1 \\ -1 \end{bmatrix}$ as 0. The first sum is equal to 0 by induction, the second one is equal to

$$v^{2r-2} \sum_{j=0}^{r-1}(-1)^j v^{j((r-1)-1)} \begin{bmatrix} r-1 \\ j \end{bmatrix},$$

hence also 0 by induction.

p. 14: The first (three line) display in "1.5:" should end with $Z^{n-1}X^r$ [not $Z^{n+1}X^r$]

†p. 17, l. −7: Actually, $E^r M_{(f)} \subset M_{(f_{-2r})}$ [not $\subset M_{(f_{2r})}$]. The rest of the proof needs to be adjusted accordingly.

†p. 19, l. 5–6: "minimal polynomial of K ..." [not k]

†p. 20, statement of 2.6: "Suppose that q ..." [not Q]

†p. 20, statement of 2.6: "integer $n \geq 0$ a simple U-module $L(n,+)$" [not $N \geq 0$ and $L(N,+)$]

†p. 22, Proof of 2.9, first sentence of paragraph 3: Replace by "We assume that M is the direct sum of its weight spaces, $M = \bigoplus_\nu M_\nu$.

†p. 24, l. 4: First character should be m_j, not v_j.

†p. 24, l. 13: Replace M' by M (twice).

†p. 24, l. 15: $\lambda^2 = q^{2(j-i)}$ should be $q^{2(j-1)}$

p. 24, l. −8: replace 2,12 by 2.12

p. 25, l. −7: replace 2.11 by 2.12

p. 26, lines 1, 2, and 4: replace p by l

†p. 26, Proof of 2.15, last line: Replace $u \in U^0$ by $u \in U_0$

p. 26, line −4: replace U^0 by U_0

p. 27, Proof of 2.18, line 2: Lemma 2.17, not 2.18

p. 29, l. −1: In the summand for $i = 0$ the product over j should run from $j = 1$ to $j = r - i - 1 = r - 1$.

p. 30, first displayed line in 2.7: $\cdots = -(q - q^{-1})^{-1}(K - K^{-1})E$.

p. 30, last displayed line in 2.7: $= \dfrac{(K - K^{-1})(q - q^{-1})}{(q - q^{-1})^2} E$.

p. 32, l. 1: replace "$E \otimes K^{-1} + 1 \otimes F$" by "$E \otimes 1 + K \otimes E$"

†p. 33, l. 14 "the way how we ..." should be "the way we ..."

†p. 35, l. −3: "U is" rather than "U in"

†p. 36, l. −4: The sentence "However, we can take ..." means in more detail: Apply (3) to M^* instead of M. So we get a U–homomorphism $(M^*)^* \otimes M^* \to k$. Now compose with the U–homomorphism $\varphi' \otimes 1 : M \otimes M^* \to (M^*)^* \otimes M^*$ with φ' as in (2). Then we get (4).

†p. 38, l. 3: "that takes a ..." rather than "that take a ..."

†p. 40, l. 14: "Each Θ_n with $n > 0$" should be $n > 1$.

p. 42, l. −6: replace 3.2(4) by 3.1(4)

p. 42, l. −2: replace 3.2(3) by 3.1(3)

p. 47, l. 1: replace 0.1(2) by 0.2(1)

p. 48, two lines before 3.10(7): $\cdots = q^{-r(r-1)}F(KF^rK^{-1})K^{r+1}$

†p. 57, l. −9: "through a homomorphisms $U = \tilde{U}/I \to 0$" has an extra "s" on homomorphism and should be mapping into k, not 0.

p. 58, lines −1 and −2: replace K_A^{-1} by $K_{\mathrm{wt}A}^{-1}$

†p. 60 and 61, eqn 3 at the top of both pages: $v_{(\beta_1,\beta_2,\ldots,\beta_{j-1},\beta_{j-1},\ldots,\beta_r)}$ should have $j+1$ instead of the second $j-1$.

p. 60, in 4.14(6): replace μ by ν

p. 61, proof, line 7: formula

†p. 61, l. −5: add after "...maximal for that": (using the ordering $\prec$ from [H], 10.1, but denoting it by $<$).

p. 64, in 4.18(7): $\cdots = q_\alpha^{i(1-r)}E_\beta$.

p. 66, two lines before 4.21(4): the left hand sides (not "right")

p. 67, lines 6–8: The sentence beginning "Theorem 4.21.a and ..." should be reformulated as follows: "The description of the bases of $\widetilde{U}^{\geq 0}$ and $\widetilde{U}^{\leq 0}$ in 4.17 implies the multiplication induces surjective linear maps $U^0 \otimes U^+ \to U^{\geq 0}$ and $U^- \otimes U^0 \to U^{\leq 0}$; Theorem 4.21.a shows that these maps are bijective."

p. 68, l. 1: replace (4) by 4.4(4).

p. 68, l. 10: replace 4.18(1) by 0.2(4).

p. 70, Proof, l. 5: replace $\pm q^a$ by $\pm q_\alpha^a$.

p. 70, l. −3: this equation should be numbered (2).

p. 71, l. -7: replace 3.10(6) by 3.5(2), replace 3.10(7) by 3.10(6).

p. 71, l. -6: replace 3.10(8) by 3.10(9).

p. 72, 5.4, l. -1: replace $M(q^{-a-2})$ by $M(q_\alpha^{-a-2})$.

p. 74, l. -7: replace I' by I.

p. 75, l. 1 of proof of 5.9: replace 5.5(1) by 5.5(4).

p. 77: the fourth line after (2) should begin "for all j such that …", (i.e, delete "all $\alpha \in \Pi$ and")

p. 77, three lines before (3): the index of $({}^\omega\widetilde{L}(\lambda'))$ should be $-\lambda' + \nu_0$, not $\lambda' - \nu_0$

p. 79, one and three lines after 5.12(7): replace $[a + r]$ by $[a + s]$

p. 79, 5.12(8): The factor after $[K_\alpha; 0]$ should be $F_{\beta_{i+1}}$

p. 80, line after (5): replace 5.1(2) by 5.1(3).

p. 81, 5.14(3): $I = (\beta_1, \beta_2, \ldots, \beta_r)$.

p. 81, l. $-10/-9$ …homomorphism of algebras $\mathbf{Q}[v, v^{-1}] \to k$ that takes v …

p. 83, l. 9: a homomorphic image of $M(\mu)$ [not of $M(\lambda)$]

p. 83, l. 10: replace 5.6.b by 5.4.b

p. 84, l. -6: replace 5.19(1) by 5.18

p. 84: Formula (3) in 5.19 should be numbered (2). And the direct sum should be over $\nu \le \lambda - \mu$.

p. 89: The alignment in (2) should be fixed.

p. 93, l. 4: add a ")" at the end.

p. 102, in 5A.8(6): The direct sum should start with $i = 0$ [not: $i = 1$]

p. 102, in 5A.8(6): The direct sum should start with $i = 0$ [not: $i = 1$]

p. 109, l. 7: replace 5.2(1) by 5.2(2).

p. 111, 6.8, l. -2: replace (2) by (1).

p. 112, l. 2 efter (1): replace 5.1(2) by 5.1(1).

p. 113, in 6.10(8): Replace F_J by E_J in the middle of the first line

p. 116, l. 1: replace K_α by $K_{\mu+\mu'-\alpha}$

p. 117, l. 5: replace 6.14(4) by 6.14(3)

p. 117, l. -1: replace 6.10(3) by 6.13(1)

p. 122, Proof, l. 2: replace 6.19(2) by 6.20(2)

p. 131, l. 3 of 7.4: replace "by Lemma 4.14" by "by 4.13(1)"

p. 134, two lines before (4): replace the second v_{i+1} by v_{i+2}

p. 138, in 7.12(2): Replace R^{rs}_{hl} (on the left hand side of the equation) by R^{rs}_{lh}

p. 138, l. 4 of proof of 7.12: $\cdots = \sum_{h,l,r,s} R^{rs}_{lh}(c'_{li} c_{hj})(u) e_r \otimes e'_s$

p. 148, l. -6: The q_α–factor should be $q_\alpha^{(i+1)(r-2m)-j(i+1)}$

p. 148, l. -5: The q_α–factor should be $q_\alpha^{(i+1)(r-2m)-(j-1)(i+1)}$

p. 148, l. -1: The q_α–factor should be $q_\alpha^{(i+1)(r-2m)-ji}$

p. 149, l. 7 and l. 9: replace $a(m-j+1)$ by $a(m-j-1)$

p. 150, l. 3/4 after the first display: delete "Proposition"
p. 152, l. 4: $= (E_\alpha u' - K_\alpha u K_\alpha^{-1} E_\alpha) T_\alpha(v)$
p. 161, l. 2 of proof of Lemma 8.21: insert $\sum_{i=0}^{r}$ after the last "="
p. 161, l. −5 of 8.21: E_α^i [not $E_\alpha^{(i)}$]
p. 162, l. 7 of 8.23: $E_\beta^a E_\alpha^b$ resp. of all $E_\alpha^a E_\beta^b$ ... [no parentheses in the exponents]
p. 164, l. 7 after (5): $q^{-(\gamma_i,\gamma_j)}$ [replace β by γ and note the sign!]
p. 167, l. 1: replace 8,26 by 8.26
p. 191, l. 2 of the proof of b): des**c**ription
p. 191, in 9.6(2): It is more logical to write $\widetilde{F}_\alpha x_{\mu-\alpha} = F_\alpha^{(2)} x_\mu = 0$.
p. 193, in 9.7(2): replace $F_\alpha^{(j)}$ by $F_\alpha^{(j-1)}$
p. 194, l. −6: replace $\mathcal{B}$ by $\mathcal{B}(\lambda)$.
p. 195, l. 10: replace v_s by v_r
p. 195, first display: both direct sums should be $\bigoplus_{i=1}^{r}$
p. 197, l. 13: $\mathcal{N} = \mathcal{M} \cap N$ [not $= \mathcal{N} \cap N$]
p. 197, l. 16: ... with $\varphi_i(v_\lambda) = v_i$ ... [not: $\varphi(v_\lambda)$]
p. 199, l. 1: replace $(\mathcal{M}, \mathcal{L})$ by $(\mathcal{M}, \mathcal{B})$
p. 200, l. 2 of Lemma 9.14: replace U_α by U^α
p. 202, l. 1: for all integers $a \geq 0$ and $b > 0$
p. 202, l. 7: this yields one inclusion in (6) [not in (10)]
p. 202, paragraph beginning with "It is now easy to get (1)–(4)": All occurrences of y, z_0 and z_1 should be overlined. (This was forgotten once for y and three times each for z_0 and z_1.)
p. 203, l. −9: replace $r \geq 0$ by $r > 0$
p. 203, 9.16(3): Left hand side should be $\widetilde{E}_\alpha^r(x \otimes b) = 0$
p. 203, 9.16(5): replace "if $i > 0$" by "if $0 < i \leq n+1$"
p. 204, six lines after (6): replace $\widetilde{E}_\alpha^2(x \otimes b) = 0$ by $\widetilde{E}_\alpha^2(x \otimes b) \neq 0$ [This is the line where we get $v \otimes \widetilde{E}_\alpha(x \otimes b)$.]
p. 205, l. 10: replace $\widetilde{E}_\alpha^j b_0$ by $\widetilde{F}_\alpha^j b_0$
p. 205, l. −3: replace 9.16 by 9.15
p. 206, l. −2 of 9.18: $\widetilde{E}_\alpha^{m+1} b' = 0$ [not $\widetilde{E}_\alpha^m b = 0$]
p. 206, l. −1 of 9.18: $= \langle \lambda, \alpha^\vee \rangle$ [not $= \langle \lambda, \alpha^\vee \rangle + 1$]
p. 208, lines −2 and −3 in paragraph beginning "c)": $\dim M > 0$ [not $\dim M < 0$]
p. 208, line −2 in paragraph beginning "c)": "... if and only if ..." [second 'if' is missing]
p. 208, l. −16: $M^* = \bigoplus_\nu (M_\nu)^*$ [not: $M = \dots$]
p. 208, l. −10: $= \psi(v)(\tau_1(u)v') =$ [not $= \psi(v)(v, \tau_1(u)v') =$]
p. 212, l. −16: this implies $\overline{q^r x} = 0$ [not $\overline{q^r y} = 0$]
p. 214, l. 7: and Lemma 9.18 [not Lemma 9.17]

p. 221, first line of 10.5: …for all $a \in \mathbf{Z}, a > 0$ [i.e., add $a > 0$]

p. 221, line after 10.5(2): and **f**or all $a, n \in \mathbf{Z}$ with $0 \le n \le a$

p. 222: Lemma 10.5 is false as stated. In order to correct it, take $x \in L(\lambda)_\mu$; we should then assume that $\langle \mu, \alpha^\vee \rangle \neq m-2$ in order for the congruence $\widetilde{E}_\alpha F_\alpha^{(m-1)} x \equiv F_\alpha^{(m-2)} x$ in (5) to hold. Similarly, for the second congruence in 10.5(9) on p. 223 to hold, we need that $\langle \mu, \alpha^\vee \rangle \ge m-i$ or $\langle \mu, \alpha^\vee \rangle < m-2i$. The mistake in the proof occurs two lines after 10.5(8) on p. 223: The equality $\widetilde{E}_\alpha F_\alpha^{(m-1)} x_0 = F_\alpha^{(m-2)} x_0$ holds only if $\langle \mu, \alpha^\vee \rangle \ge m-i$ or $\langle \mu, \alpha^\vee \rangle < m-2i$ (in the second case because both sides are 0). — The correction makes necessary a small change in the proof of Lemma 10.6. We apply there Lemma 10.5 with $x = yv_\lambda \in L(\lambda)_{\lambda-\nu+n\alpha}$ and $m = n+1$. So we need to know that $\langle \lambda - \nu + n\alpha, \alpha^\vee \rangle \neq n-1$, i.e., that $\langle \lambda, \alpha^\vee \rangle \neq \langle \nu, \alpha^\vee \rangle - (n+1)$. That is no problem since we later assume that $\langle \lambda, \alpha^\vee \rangle \ge \langle \nu, \alpha^\vee \rangle + n$. So we just have to state that assumption somewhat earlier on in the proof.

p. 222, l. −1: $\begin{bmatrix} m-i+j \\ j \end{bmatrix}_\alpha$ [j, not i, in second row]

p. 224, two lines before (4): $q_\alpha^{i(\langle \lambda-\nu+n\alpha, \alpha^\vee \rangle + i + 1)}$ in the numerator [additional n]

p. 224, one line before (4): $\lambda - \nu + (i+n)\alpha$

p. 224, 10.6(4): $q_\alpha^{i(2\langle \lambda-\nu, \alpha^\vee \rangle + 2i + 3n)}$ in the numerator [$3n$ instead of $4-n$]

p. 224, last display in 10.6: first factor should be $q_\alpha^{i(2\langle \lambda-\nu, \alpha^\vee \rangle + 2i + 3n)}$ [as in previous correction], the middle factor on the right hand side should be $q_\alpha^{i(\langle \lambda, \alpha^\vee \rangle - \langle \nu, \alpha^\vee \rangle + 2i + 4n + 1)}$

p. 224, l. −5: …made at the end …[delete 'in']

p. 224, l. −3: replace Proposition by Theorem

p. 228, l. 8 replace Lemma 10.9 by Proposition 10.9

p. 229, first line of Remark: $(\mathcal{L}(\infty), \mathcal{B}(\infty))$

p. 230, Prop. 10.14: "$\overline{\varphi}_\lambda$ induces" [NOT: $\overline{\varphi}_\lambda(b)$]

p. 231, 10.16(1): right hand side is $U_\nu^+ K_\nu$

p. 232, l. −3: replace 6.16(6) by 6.15(6)

p. 233, l. −3 of 10.17: replace $r_\alpha(x)$ by $r'_\alpha(x)$

p. 238, 11.1(5): Add an index α to the Gaussian binomial coefficient

p. 238, 11.1(6): Replace both μ by ν

p. 239, Lemma 11.2.b: replace *with* $r'_\alpha(u) = 0$ *for all* n by *with* $r'_\alpha(u_n) = 0$ *for all* n

p. 245, first line after 11.8(2): Replace $U_{\mathbf{Z}}^0$ by $U_{\mathbf{Z}}^-$

p. 245, Proposition 11.9: The word 'Proposition' is in a wrong font

p. 247, l. −2 in 11.10: add a ")" after $q\mathcal{L}(\infty)_{-\nu+n\alpha}$

p. 249, three lines after 11.12(2): 11.10.c [not 11.10.b]

p. 250, two lines before 11.13(4): $e_\alpha(d) = 0$ [not > 0]

p. 251, l. 3: $f_\alpha(\overline{\varphi}_\lambda(b))$ [not (d)]

p. 251, first line after 11.13(6): $S = \{G^\alpha(b)v_\lambda \mid b \in \mathcal{B}(\infty)_{-\nu}, \overline{\varphi}_\lambda(b) \neq 0, e_\alpha(b) \ge n\}$

p. 251, first line after 11.14(1): $\varphi_\lambda(b) = \widetilde{F}_{\alpha_1} \dots \widetilde{F}_{\alpha_{r-1}} \widetilde{F}_\beta \varphi_\lambda(1) = 0$

p. 252, l. 12: ...yields 11.10.b.

p. 252, two lines before 11.15(1): $S = \{G(b)v_\lambda \mid b \in \mathcal{B}(\infty)_{-\nu}, \overline{\varphi}_\lambda(b) \neq 0\}$

p. 254, l. −3: $= F_\alpha^{(r-i)} F_{\alpha+\beta}^{(i)} F_\beta^{(s-i)} + \dots$ AND: add an index α to the Gaussian binomial coefficient

p. 255, l. 3: $\equiv F_\alpha^{(r-i)} F_{\alpha+\beta}^{(i)} F_\beta^{(s-i)}$

p. 255, l. 5: $\equiv F_\alpha^{(r-i)} F_\beta^{(s)} F_\alpha^{(i)}$

p. 255, l. 6: So we see that $F_\alpha^{(r-i)} F_\beta^{(s)} F_\alpha^{(i)} \dots$

p. 255, Proposition 11.18: $\tau G(b) = G(\tau b)$

p. 255, line 2 of the proof of 11.18: $\tau G(b) = G(\tau b)$

p. 255, line 7 of the proof of 11.18: $r'_\alpha(u'_j) = 0$

p. 255, l. −8: since $e_\alpha(b') = 0$ [NOT: > 0]

p. 257, four lines before 11.19(3): $(U_{\mathbf{Z}}^-)_{-\mu} \otimes \mathbf{C}$

p. 260, ref. Jimbo 2: replace "q–difference analogue" by "q–analogue"

p. 261, ref. Rudakov & Shafarevich: Delete the comma at the end

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